Name- MAX
Math 213, May 2024, Test 2

You may use your page of notes / calculator / writing instruments on this test.

1. (21 pts) A metal plate is situated in the xy plane and occupies the rectangle $0 \leq x \leq 10,\ 0 \leq y \leq 8$ where x and y are measured in centimeters. The temperature at the point (x, y) in the plate is $T(x, y)$, where T is measured in degrees Celsius. Temperatures at equally spaced points on the plate were measured and recorded. A portion of those measurements is shown in the table below.

	y=2	y=4	y=6
x=4	60 C	64	62
x=6	84	76	72
x=8	100	90	82

   a. Estimate the values of the partial derivatives $T_x(6, 4)$ and $T_y(6, 4)$. Give *units* with your answers.
   b. Using these partial derivative estimates, write down the linear approximatiof $T(x, y)$ for points near $(6, 4)$.
   c. Using your linear approximation, estimate the value of $T(5, 3)$.

2. (9) You are told that a certain function, $f(x, y)$ has these partial derivatives:

$$\frac{\partial f}{\partial x} = 3y^2 + 2x + \ln(x * y) + 37 \tag{1}$$

$$\frac{\partial f}{\partial y} = 6xy + \frac{x}{y} + \cos(y^2) \tag{2}$$

Carry out Clairaut's test to decide if $f(x, y)$ is a continuous function or not. Show the results of your test, and state your conclusion about whether it is continuous or not.

3. (12) Consider the function

$$f(x, y) = \frac{x + y}{|x| + |y|} \tag{5}$$

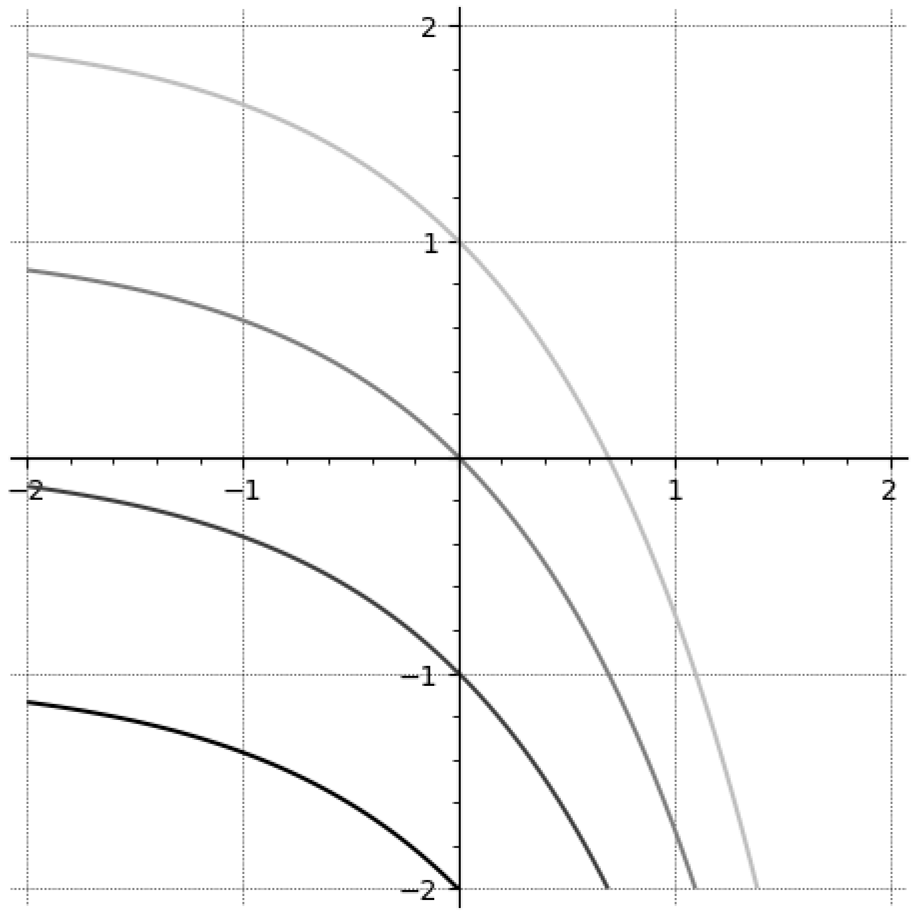
(a) Evaluate the function at the following locations:

   i. $f(0.1, 0.1)$
   ii. $f(0.1, -0.1)$
   iii. $f(-0.1, 0.1)$
   iv. $f(-0.1, -0.1)$

(b) Either find $\lim_{(x,y)\to(0,0)} f(x, y)$ or show that it doesn't exist.

4. (24) Several level curves for the function $f(x, y) = e^x + y$ are shown below.

a. What is the value of $k = f(x, y)$ on each of the level curves ($z = k$ traces) shown? Label each level curve with the value of k *on* that level curve.



b. What is the formula, $y = f(x)$, for the level curve shown that passes through the point $(0, 1)$.

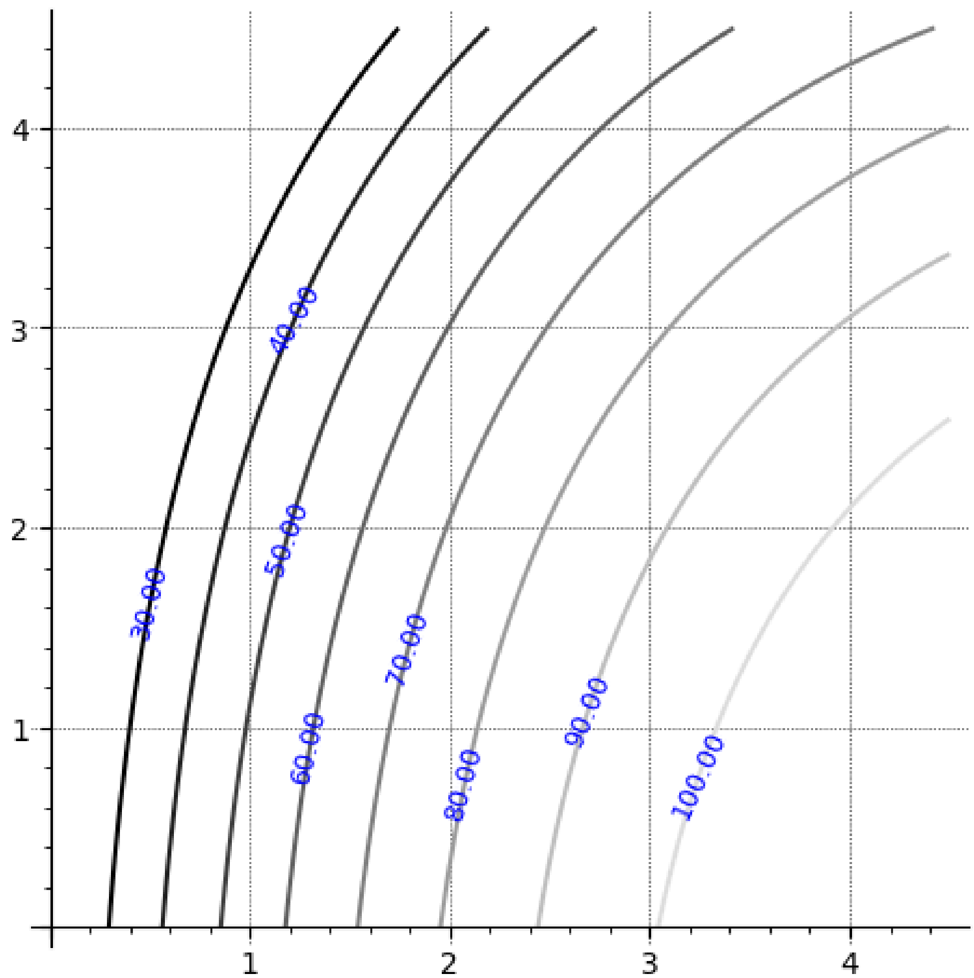
c. At the point $(0.7, 0)$, calculate, then draw and label the vector $\vec{\nabla} f$. Be fairly accurate as to the direction and length of this vector.

d. On the graph, draw and label a vector that starts at $(0.7, 0)$ and points in a direction in which $f(x, y)$ remains constant. What are the components of the vector you found?

e. Find the directional derivative of $f(x, y)$ at the point (0.7, 0) in the direction of $\vec{\mathbf{u}} = 3\hat{\imath} + 4\hat{\jmath}$.

f. What is the maximum possible value of *any* directional derivative at $(0.7, 0)$?

5. (15) The following is a contour plot of a function $z = f(x, y)$. Values $z = k$ are indicated on the plot.



Estimate from the contour plot the values of these partial derivatives at $(2, 3)$:

$f_x = ?$

$f_y = ?$

Determine whether the following derivatives are positive or negative or 0 at the point $(2, 3)$. *Explain* your answers.

f_{xx}

f_{yy}

f_{xy}

6. (12)

   a. Find A and B so that $f(x,y) = x^2 + Ax + y^2 + B$ has a critical point at (7,0) with z-coordinate 5.

   b. Is the critical point a maximum, a minimum, or neither?

7. (20) Evaluate the integral below by hand, showing all steps in the computation, including antiderivatives. Your answer should be a single fraction or decimal number (to the nearest thousand'th).

$$\int_0^2 \int_{y=-1}^1 (3x + y^2)\, dy\, dx \tag{6}$$

8. (17) In the figure, the contour plot of $f(x, y) = x^2 + xy + y^2$ is shown. (Dark blue are regions where f is low and in orange-yellow-colored regions, f is high.) Also, a path (in red) given by $g(x, y) = y - x^2 = -3$ is drawn in.
   a. Find the coordinates of all the critical points of the function $f(x, y)$. For each one, state whether it is a maximum, a minimum, or neither. If a maximum or minimum, state whether it is a global or local one. Mark the position of any that would appear within the bounds of the diagram.
   b. The red line is a path through the "landscape" of the surface, f, going up and down, according to the values of f as indicated by the contour plot. Estimate the position(s) of any critical points $f(x, y)$ along the red path. Mark and label each one on the path on the diagram. Determine whether each is local maxima, local minima, or neither and write down your determination for your labelled points(s).